The Weight Reduction of Charged Capacitors, Charge-Mass Interaction, and Einstein's Unification

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ABSTRACT

The Biefeld-Brown (B-B) effect consists of two parts: 1) the initial thrust is due to the electric potential that moves the electrons to the positive post; and 2) the subsequent lift is due to the separate concentration of the positive and the negative charges. The weight reduction of a charged capacitor is due to a repulsive charge-mass interaction, which is normally cancelled by the attractive current-mass interaction. In a charged capacitor, some electrons initially moving in the orbits become statically concentrated and thus a net repulsive force is exhibited. Based on observations, it is concluded that a repulsive charge-mass interaction is proportional to the charge density square and diminishes faster than the attractive gravitational force, and that the current-mass force is perpendicular to the current. This charge-mass interaction is crucial to establish the unification of electromagnetism and gravitation. To confirm general relativity further, experimental verification of the details of this mass-charge repulsive force is recommended. Moreover, general relativity implies that the photons must include gravitational energy and this explains that experiments show that the photonic energy is equivalent to mass although the electromagnetic energy-stress tensor is traceless. In general relativity, it is crucial to understand nonlinear mathematics and that the Einstein equation has no bounded dynamic solutions. However, due to following Einstein's errors, theorists failed in understanding these and ignored experimental facts on repulsive gravitation. Since the charge-mass interaction occurs in many areas of physics, Einstein's unification is potentially another revolution in physics. Moreover, the existence of a repulsive gravitation implies the necessity of re-justifying anew the speculation of black holes.

Key words
dynamic solution; charge-mass interaction; repulsive gravitation; current-mass interaction; E = mc^2.
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1. Introduction

Thomas Townsend Brown [1] developed an electric capacitor device that he termed a gravitator because he believed that somehow the gravitational field would become affected by the plate high voltage charge. When energized with up to 150,000 volts of direct current, Brown's gravitator developed a thrust in the direction of its positively charged end. When oriented upright on a scale and energized, it proceeded to gain or lose that amount of weight depending upon the charge polarity. It became lighter when the positive end faced up and heavier when the negative end faced up. He was later joined by Paul A. Biefeld, at Dennison University. This effect is now known as Biefeld-Brown (B-B) effect [1], and is referred to as due to the electrogravitics, which is defined by Valone [2] as "electricity used to create a force that depends upon an object's mass".

A problem of the B-B effect is that this effect cannot be explained with current theories. For instance, according to Einstein's $E = mc^2$ [3], a charged capacitor should increase its weight, but it actually decreases its weight [1, 2]. This is also surprisingly related to the fact that the Einstein equation does not have dynamic solutions [4]. However, some tried to work out a theory within the current theoretical framework with imaginative but inadequately justified assumptions, instead of working out a formula based on observations and/or with established theories [1, 2]. However, a result is that many theorists incorrectly regarded the B-B effect as just experimental error. (On the other hand, they all failed to see Einstein's errors, and follow Einstein's step to wrong results [4].) Perhaps, it is time to remind theorists that the current physical laws are originally obtained from and tested by observations. In this paper, we try to take this established approach.

2. Some Established Facts of the B-B Effects

The established facts are as follows:

1) When energized with up to 150,000 volts of direct current, Brown's gravitator developed a thrust in the direction of its positively charged end [1, 2].

2) Brown [1] suspended a gravitator with two wires, and forming a pendulum that is immersed in a tank of oil. When energized, the pendulum would swing toward the gravitator's positive post. He noted that after the pendulum reached the maximum amplitude of its swing, then, even while he maintained the high-voltage potential, his pendulum would gradually return to its plumb position. The thrust can be repeated after the gravitator takes a rest. Moreover, when the duration of the gravitic impulse had been greater, more time was needed off-line to allow the gravitator to refresh itself.

3) The lifter (a light capacitor) is able to lift its own weight plus a payload after being charged with a high voltage (about 40 kilovolts), but without continuous supply of electric energy. A lifter could get to work by charging the wire to either a positive or a negative potential. It has been determined that the thrust is not due to ion wind effects [1, 2]. Thus, thrust is generated by changing something of the lifter with one high voltage charge.

From the above, apparently the B-B effect consists of two parts: 1) the initial thrust is due to the electric potential to move the electrons to the positive post; and 2) the subsequent lift is due to the concentration of the positive and negative charges separately. The second point is the essence of electrogravitics. Note that the lifting is against the attraction of earth and thus the effect is not clearly observed in the pendulum case.

3. The Charged Capacitors and the Charge-Mass Interaction

It is known that a charged capacitor reduces its weight. In a charged capacitor, the only change is the state of motion of some electrons that have become statically concentrated instead of moving in orbits. Then, a repulsive force appears. Since such a force does not appear before, it is clear that such a force was cancelled out by the force created by the motion of the electrons. In other words, the repulsive force generated by the charges of protons and the electrons was cancelled by the force generated by the motion of the initially moving charges of the electrons.

However, this repulsive force cannot be proportional to the charge density although it is acting on charged particles because of their charge. We have equal numbers of negatively charged particles (electrons) and positively charged particles (protons) and that would lead to the cancellation of the forces generated by particles of different charges. However, if such a force is proportional to the charge density square, then these two kinds of forces would be added up, instead of cancelled out. Moreover, since the lifter has a limited height, one should expect that this repulsive gravitational force would diminish faster than the attractive gravitational force. Thus, if we assume that the force is proportional to mass as usual, the static charge-mass interaction would be a repulsive force between particles with charge density $D_z$ and another particle of mass $m$ would have the following form,

$$F = K m D_z^2 r^n$$

where $n > 2$, \( n = 3 \) is the distance between the two particles, and $K$ is the coupling constant. In formula (1), the coupling constant $K$ and $n$ the power of $r$ can be determined by experiments. The simplest case is $n = 3$.

Formula (1) is derived from the observations with common physical sense. The experimental facts are that the charged capacitors have reduced weight. This reduced weight is caused by a repulsive force that can lift a device. Thus, it can be used as guidance for related discussions. Such an approach has to be taken because we recognized that there is little in common between the charge-mass interaction and current theories. This approach is different from the derivations from theories with very imaginative assumptions without adequate justifications [5, 6].

Theorists who are familiar with general relativity would immediately recognized that a term similar to Eq. (1) appears in the Reissner-Nordstrom metric [7] for a particle with charge $q$ and mass $M$, as follows:

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)dt^2 - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1}dr^2 - r^2d\Omega^2. \quad (2)$$

The static force that acts on a test particle $P$ with mass $m$ for the first order approximation is

$$-m\frac{M}{r^2} + m\frac{q^2}{r} \quad (3)$$

since $g' = \frac{-1}{r^2}$. Note that the second term is a repulsive force due to the static charge-mass interaction. According to the reaction force being equal to but in the opposite direction of the acting force, the test particle $P$ must create a field $m/r^2$ that couples to $q^2$. This would mean that unification between electromagnetism and gravitation is necessary [8]. In the first term of (3), $M$ is the inertial mass of the charged particle.

However, the newly discovered force was over-looked until 1997 [9] after it was recognized that the mass is not equivalent to electric energy [10]. This overlooking was due to two misconceptions: 1) Gravity is always attractive; 2) $E = mc^2$ was incorrectly considered as unconditional [3]. The non-existence of a dynamic solution for the Einstein equation leads to the discovery that there must be different coupling signs for the dynamic case [10]. This non-uniqueness of couplings leads to the investigation that the charge-mass interaction was discovered [10-12].

The experiments on a charged ball confirm the existence of a repulsive charge-mass interaction (3) [13], and this would confirm the unification of electromagnetism and gravitation. Einstein over-looked the coupling of charge square in the five-dimensional theory [14] because he believed, unlike Maxwell, that a new interaction should not be created. Since formula (3) is generated by general relativity and thus is also a crucial test for general relativity. However, Einstein and his peer theorists have mistaken that the electromagnetic energy was equivalent to mass [15].

5. The Charge-Mass Interaction and Five-dimensional Theory

Force (3) to particle $Q$ is beyond current theoretical framework of gravitation+ magnetism. However, although the charge square coupling is beyond Einstein’s general relativity, in the five-dimensional theory [16] the geodesic equation would include the coupling of $q^2$. As Kaluza [17] proposed, a five-dimensional geodesic is

$$\frac{d}{dr} \left( g_{ik} \frac{dx^k}{dr} \right) + \Gamma_{ik}^j \frac{dx^i}{dr} \frac{dx^j}{dr} = 0, \quad \text{where} \quad \Gamma_{ik}^j = \frac{1}{2} \left( \frac{\partial g_{ij}}{\partial x^k} + \frac{\partial g_{ik}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^i} \right) g^{ij}. \quad (4)$$

and $ds^2 = g_{ij}dx^i dx^j$, and $g_{ij}$ ($\mu, \nu = 0, 1, 2, 3, 5$) are metric elements of a five-dimensional space.

After some algebraic calculation, the geodesic equation (4) can be represented as follows:

$$\frac{d}{ds} \left( g_{sk} \frac{dx^k}{ds} + \frac{1}{2} g_{ss} \frac{dx^k}{ds} \frac{dx^l}{ds} \right) = \Gamma_{sk}^{il} \frac{dx^k}{ds} \frac{dx^l}{ds} - \frac{1}{2} g_{ss} \frac{dx^k}{ds} \frac{dx^l}{ds} + \Gamma_{sk}^{s} \frac{dx^l}{ds} \frac{dx^k}{ds}, \quad (4a)$$

and

$$\frac{d}{ds} \left( g_{sk} \frac{dx^k}{ds} + \frac{1}{2} g_{ss} \frac{dx^k}{ds} \frac{dx^l}{ds} \right) = \Gamma_{sk}^{il} \frac{dx^k}{ds} \frac{dx^l}{ds} - \frac{1}{2} g_{ss} \frac{dx^k}{ds} \frac{dx^l}{ds} + \Gamma_{sk}^{s} \frac{dx^l}{ds} \frac{dx^k}{ds}. \quad (4b)$$

The electromagnetism is included by assuming $g_{\mu\nu} = KA_{\mu}(\mu, \nu = 0, 1, 2, 3)$ where $A_{\mu}$ is the potential, and $K$ is a constant.

In Kaluza’s theory [17], the five variables are reduced to only four variables. Einstein and Pauli [18] assumed that all the “extra” metric elements are negligible, and thus failed to see the possibility of including new features. In the theory of Lo et al. [16], because of less restrictions, it can accommodate the radiation reaction force and the new charge-mass interaction.

Instead of $s$, $\tau$ is used in (4), the Lorentz force suggests

$$\frac{q}{Me^2} \left( \frac{\partial A_k}{\partial x^k} - \frac{\partial A_k}{\partial x^l} \right) \frac{dx^l}{d\tau} = \left( \frac{\partial g_{ik}}{\partial x^l} - \frac{\partial g_{ik}}{\partial x^l} \right) dx^l dt$$

Thus,
Currently the electromagnetic field in a capacitor would cancel out, there would be no forces in the perpendicular direction of $K^\parallel$.

The existence of the current would create the magnetic field. Thus, they also measured the existence of the current which appeared to make a capacitor reduce its mass repulsive potential would exist for a metric based on the charge $\mathbf{q}$. A repulsive force would be cancelled by the current $k$.

Due to the fact that a charged capacitor has reduced mass repulsive force would be cancelled by the effect of the charge $\mathbf{q}$, the five-dimensional theory would solve the problem of charge-mass interaction completely although eq. (1) for the case of capacitor is rather heuristic. To address the issue of charge-mass interaction one must have a solid theoretical ground, and thus the details of formula (3) should be completely tested with experiments.


If the electric energy leads to a repulsive force toward a mass according to general relativity, the magnetic energy would lead to an attractive force from a current toward a mass [19, 20]. Due to the fact that a charged capacitor has reduced weight, it is necessary to have the current-mass interaction to be cancelled out by the effect of the charge-mass interaction. In other words, the existence of the current-mass attractive force would solve a puzzle, i.e., why a charged capacitor attracts a mass. Physically the static charge-mass interaction should not be subject to electromagnetic screen since it is proportional to charge square. However, since currently the electromagnetic field in a capacitor would cancel out, there would be no charge-mass interaction beyond the capacitor. However, one may hope that the five-dimensional theory would solve the problem of charge-mass interaction completely although eq. (1) for the case of capacitor is rather heuristic. To address the issue of charge-mass interaction one must have a solid theoretical ground, and thus the details of formula (3) should be completely tested with experiments.
the surface. (This weight reduction is directional.) Note that our explanation on the weight reduction of alternative current is very different from some current theories that treat the weight reduction as directly due to the alternative current [5].

One may ask what is the formula for the current-mass force. Unlike the charge-mass repulsive force, which can be derived from general relativity; this general force would be beyond general relativity since a current-mass interaction would involve the acceleration of a charge, this force would be time-dependent and generates electromagnetic radiation. Moreover, when the radiation is involved, the radiation reaction force and the variable of the fifth dimension must be considered [16]. Thus, we are not ready to derive the current-mass interaction yet.

Nevertheless, we may assume that, for a charged capacitor, the resulting force is the interaction of net macroscopic charges with the mass [22]. The irradiated ball has the extra electrons, and thus reduced its weight [13]. A spinning ring of superconducting material has the attractive currents that are attractive to the earth. This also explains a predicted phenomenon, which is also reported by Liu [23] that it takes time for a capacitor to recover its weight after being discharged [22]. This was observed by Liu because his rolled-up capacitors keep heat better. A discharged capacitor needs time to dissipate the heat generated by discharging, and the motion of its charges would accordingly recover to normal.

7. Experimental Verification of the Mass-Charge Repulsive Force

The repulsive force in metric (3) can be detected with a neutral mass. To see the repulsive effect, one must have

\[
\frac{1}{2} \frac{\partial}{\partial r} \left( 1 - \frac{2M}{r} + \frac{q^2}{r^2} \right) = \frac{M}{r^3} - \frac{q^2}{r^3} < 0
\]

(9)

Thus, repulsive gravity would be observed at \( q^2/M > r \). For the electron the repulsive gravity would exist only inside the classical electron radius \( r_0 = 2.817 \times 10^{-13} \text{ cm} \). Thus, it would be very difficult to test a single charged particle.

However, for a charged metal ball with mass \( M \) and charge \( Q \), the formula is \( 0 > M/R^2 - Q^2/R^2 \), where \( R \) is the distance from the center of the ball [24]. The attractive effect in gravity is proportional to mass related to the number of electrons, but the repulsive effect in gravity is proportional to square of the number of electrons. Thus, when the electrons are accumulated numerous enough in a metal ball, the effect of repulsive gravity will be shown in a macroscopic distance.

Consider that \( Q \) and \( M \) is consist of \( N \) electrons, i.e., \( Q = Ne \), \( M = Nm_e + M_0 \), where \( M_0 \) is the mass of the metal ball, \( m_e \) and \( e \) are the mass and charge of an electron. To have sufficient electrons, the necessary condition is

\[
N > \frac{R}{r_0} \quad (\text{or} \quad 0 > \left[ \frac{Nm_e}{R^2} - \frac{N^2e^2}{R^3} \right] m_p) \quad \text{where} \quad r_0 = \frac{e^2}{m_e c^2} = 2.817 \times 10^{-13} \text{cm}. \quad (10)
\]

For example, if \( R = 10 \text{ cm} \), then it requires \( N > 3.550 \times 10^9 \). Thus \( Q = 5.683 \times 10^7 \text{ Coulomb} \). Then, one would see the reduction of attractive gravitation when condition (10) is satisfied. For this case, the repulsive force is

\[
\frac{Q^2m_p}{R^3}
\]

(11)

where \( m_p \) is the mass of the testing particle \( P \), and the total force is

\[
\left( \frac{M_0 + Nm_e}{R^2} - \frac{N^2e^2}{R^3} \right) m_p . \quad (12)
\]

When condition (9) is satisfied for a certain \( R \), the repulsive effect will be observed; and the charged ball would even lift up as the charge increases sufficiently. Currently, the majority of theorists failed gravity by following Einstein’s error. The verification of this formula is also disproves the equivalence between mass and electric energy.

Moreover, before the repulsive effect is detected, for the reduction of attractive gravity to be seen requires only \( N > R/r_0 \). The relation (10) is a much easier condition to be satisfied. This is why Tsipenyuk & Andreev [13] observed a reduction of weight after a metal ball is sufficiently charged.

However, since the repulsive force is very small, the interference of electricity would be comparatively large. Thus, it would be desirable to screen the electromagnetic effects out. The modern capacitor is such a piece of simple equipment. When a capacitor is charged, it separates the electron from the atomic nucleus or polarizes the atom, but there is no change of mass due to increase of charged particles. Thus, after charged, the fact that capacitor would have less weight is a proof that the electric energy is not equivalent to mass. In some cases when such a capacitor is charged with high electric potential, it can even lift up [1, 2, 5]. This simple experiment would confirm also the mass-charge repulsive force, and thus the theoretical unification of electromagnetism and gravitation in term of a five-dimensional theory.

One may ask whether the lighter weight of a capacitor after charged could be due to a decrease of mass. Such a speculation is ruled out. Inside a capacitor the increased energy due to being charged would not be pure electromagnetic energy such that, for the total internal energy, Einstein’s formula is valid.
In the case of a charged capacitor, the repulsive force would be proportional to the potential square, \( V^2 \) where \( V \) is the electric potential difference of the capacitor \( (Q = CV, \ C \) is the capacitance). This has been verified by the experiments of Musha [6, 25]. Thus, the factor of charge density square in heuristic Eq. (1) is correct. It remains to verify the space dependence. However, even the \( 1/r^2 \) factor in Eq. (1) is verified, the calculation would still depend on the detailed modeling (see Appendix A). In the B-B effect, the initial thrust is directional decided by the electric field. However, the weight reduction effect for charged capacitors is not directional and it stays if the potential does not change. This is verified by Liu [23], who measured the effect of weight reduction with the roll-up capacitors.

8. Conclusions and Discussions

In this paper, the most important conclusion is the confirmation of the repulsive charge-mass interaction from general relativity. This repulsive gravitation had been over-looked because \( E = mc^2 \) has been misinterpreted by Einstein [3] as unconditionally valid. However, it is clear that the repulsive force of gravity depends on the square of charge density [25] although derivations to specifics are incomplete because of the lack of detailed modeling (see Appendix A). The improvements for such shortcomings need the input of some accurate experimental data as well as better theoretical understanding. Nevertheless, the existence of lifters [1, 2] unequivocally announces the existence of repulsive gravitation. Then, it is easier to see that the progress of general relativity is blocked by Einstein's own errors.

On the other hand, invalid derivations following Einstein's errors [4], were not discovered. Two major problems of Einstein's theory are that he failed to see that there is no bounded dynamic solution for the 1915 Einstein equation [10, 26] as Gullstrand suspected [27] and \( E = mc^2 \) is not valid for the electromagnetic energy [9]. Thus, the conjecture for the existence of the black holes must be justified anew because the assumptions [20], validity of the Einstein equation for the dynamic case and gravitation being always attractive are incorrect.

Now, important conclusions from the weight reductions of charged capacitors are: 1) \( E = mc^2 \) is only conditionally valid, and the electromagnetic energy is not equivalent to mass. 2) Einstein's conjecture of unification is established; 3) Einstein's general relativity is valid only for the static and stable cases, but is invalid for the dynamic case [10, 26]. It remains to be rectified and completed in at least two aspects: a) The exact form of the gravitational energy-stress tensor is not known; and b) The radiation reaction force is also not known [4]. Note that, due to the influence of a radiation reaction force, the viewpoint of considering general relativity as a theory of geometry is invalid. Moreover, since the photons include gravitational energy [28], the unification of gravitation and electromagnetism is necessary. To this end, a potentially very strong candidate is a five-dimensional theory [19].

Moreover, the space-time singularity theorems of Hawking and Penrose are proven to be irrelevant to physics since they are based on an invalid assumption, the non-existence of the antigravity coupling [4]. Thus, their contribution to physics is essentially zero if not negative. The positive theorem of Yau and Schoen is misleading [29] because the invalid assumption was used. These errors follow Einstein's mistake of believing in the existence of dynamic solutions for his equation.

According to the position on this issue, journals can be divided into three categories, namely: 1) Agreeing with Gullstrand [27] Chairman of the Nobel Committee for Physics (1922-1929) that there is no dynamic solution for the Einstein equation; 2) Disagreeing with Gullstrand that the Einstein equation has dynamic solutions, but failed to provide even one example; 3) Unable to decide whether the Einstein equation has any dynamic solution. Currently the Physical Review, Proceeding of the Royal Society A, Classical and Quantum Gravity, Gravitation and General Relativity, etc. all are against Gullstrand [27], and thus still are the obstacles for the progress in physics. On the other hand, journals such as the Astrophysics Journal, Physics Essays, etc. have committed to my position by publishing my paper [10-12]. However, some journals such as Annalen der Physik prefer not to fix their position on this. Thus, currently some still failed in understanding gravity and ignore experimental facts such as the existence of lifters [1, 2].

Einstein has shown that the photons are equivalent to mass [15] and this is supported by the experimental fact that a \( \pi^0 \) meson would decay into two light rays \( (\pi_0 \rightarrow \gamma + \gamma) \). Thus, Einstein had mistaken that the electromagnetic energy is equivalent to mass. This is impossible because the electromagnetic energy-stress tensor is traceless, and the addition of electromagnetic energies is still an electromagnetic energy. This conflict is finally solved by recognizing that the photons include not only electromagnetic energy, but also gravitational energy [28, 30]. In other words, Einstein's initial proposal of photons is inadequate (see Appendix B). This may be the main reason that QED must be renormalized. Moreover, general relativity is incomplete because the gravitational radiation reaction force is not included [31].

Einstein [3] claimed that a piece of heated-up metal would have increased weight. Experimentally, however, six kinds of heated-up metals have reduced weight [22]. Because a charged capacitor has reduced weight, it will fall slower than other neutral objects [32]. Thus, Einstein's unification would affect many areas of physics. The charge-mass interaction appears in many areas of physics, but its effects have not been considered. Thus, not only gravitational theories, but also other related physicists must be reviewed carefully. Moreover, the charge-mass interaction is not only crucial for the unification of electromagnetism and gravitation, but also has useful applications because it would lead to a new way of observation.

Among many potential applications in astrophysics, it is the charge-mass interaction that provides an explanation of NASA's discovery of the Space-Probe Pioneer Anomaly [33], and a new technology of detecting matters under ground and/or under the water [25]. Thus, the discovery of the charge-mass interaction is a big step of progress in modern physics. However, some physicists have irrationally denied such repulsive gravitation because they would not admit their own errors.
Einstein’s formula \( E = mc^2 \) is only conditionally valid and its misinterpretation is the source of many errors in general relativity. Obviously, the current-mass interaction is important in Einstein’s unification. However, it has not been adequately addressed, and thus some issues cannot be answered. It is hope that such work can be completed in the future. It is interesting that Einstein is still the number one theorist in physics after the rectification of his theory although his general relativity has many problems of misleading in physics. Another lesson is that all human institutes are not perfect and can make scientific errors [29, 31, 34, 35]. We respect Nobel laureates for their achievements. However, they could also be incorrect in physics [35-37]. Thus experiments together with careful analysis are still the most reliable source of information.

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Appendix A: Some Remarks on the Charge-Mass Repulsive Force

The charge-mass repulsive force between a point charge \( q \) and a point mass \( m \) is,

\[
F = \frac{q^2 m}{r^2} \tag{A1}
\]

would behave very differently from a Newtonian force, which is inverse proportional to the square of the distance \( r \).

To illustrate this, let us calculate the force between a charge \( q \) and a mass \( M \) whose density \( \rho \) is uniformly distributed in a sphere of radius \( R \). In a coordinate \(( r, \theta, \phi) \), let \( s \) be the distance between the center of the sphere, located at \((0, 0, 0) \) and the charge \( q \) is at \((s, 0, 0) \). Let \( l \) be the distance between the charge \( q \) and a mass \( dm \) at a point \((r, \theta, \phi) \). Then we have

\[
F = s^2 + l^2 - 2sr \cos \theta, \quad \text{and} \quad dm = \rho(rdr)drd(\sin \theta)d\phi = \rho r^2drd(-\cos \theta)d\phi, \tag{A2}
\]

where \( \rho \) is the mass density. We calculate the force potential, which is locally proportional to \( l^2 \), and have

\[
\int_0^\pi \frac{d(-\cos \theta)}{s^2 + r^2 - 2sr \cos \theta} = -\frac{1}{2sr} \ln(s^2 + r^2 - 2sr \cos \theta) \Big|_0^\pi = -\frac{1}{sr} \ln\left(\frac{s + r}{s - r}\right) \tag{A3}
\]

Then the total potential \( V(s, R) \) would be

\[
2V(s, R) = \rho \int_0^R 2\pi r^2dr \ln\left(\frac{s + r}{s - r}\right) = \frac{2\pi}{s} \int_0^R rdr \ln\left(\frac{s + r}{s - r}\right) = \frac{2\pi}{s} \int_0^R rdr s^2 + r^2 - 2sr \cos \theta = \frac{3}{s} \ln\left(\frac{s + r}{s - r}\right) \tag{A4}
\]

Note that \( \left[1 + \frac{3}{5} + \frac{3}{7} + \ldots \right] = \frac{3}{2} \), and \( M = \rho \left[\frac{4\pi R^3}{3}\right] \).

Thus, the repulsive force between the charge \( q \) and the mass \( M \) is

\[
2F(s, R) = 2q^2V(s, R) = \frac{q^2 M}{s} \left[1 + \frac{3}{5} + \frac{3}{7} + \ldots \right] \tag{A5}
\]

Note that \( -\frac{3}{2s^3} \left[1 + \sum_{n=1}^\infty \frac{1}{2n + 1}\right] \) does not converge.

The close forms of the repulsive potential and the repulsive force are,

\[
V(s, R) = \frac{\pi}{s} \int_0^R rdr \ln\left(\frac{s + r}{s - r}\right) = \frac{\pi}{s} \left(\frac{R^2 - s^2}{2} \ln\left(\frac{s + R}{s - R}\right) + sR\right) \tag{A6}
\]
\[ F(s, R) = q^2 V_2(s, R) = \cdot q^2 s^2 \pi \left( \left[ 1 - \left( \frac{R}{s} \right)^2 \right] \frac{1}{2} \ln\left( \frac{s + R}{s - R} \right) \right) \]  

\[(A7)\]

Now, it is clear that the repulsive force on the charge \( q \) is sensitive to the surrounding of the charge. For a charged object, the repulsive force is sensitive to the arrangements of the charges. Note that the dependence of \( r^{-2} \ln (A1) \) is derived from general relativity. Thus, this force provides another test of general relativity. However, it would be difficult to calculate the effects of this force without the necessary detailed information.

**Appendix B: The Electromagnetic Wave, Photons and Anti-gravity Coupling**

For the validity of the calculation on light bending, it is necessary that an electromagnetic wave would generate a negligible gravitational wave because this was implicitly assumed in the calculation of the bending of the light ray.

For the electromagnetic wave as the source, the related Einstein equation must be modified [28] as the following:

\[
G_{\mu\nu} = -K(T(E)_{\mu\nu} - T(P)_{\mu\nu}), \quad \text{and} \quad T_{\mu\nu} = -T(g)_{\mu\nu} = T(E)_{\mu\nu} + T(P)_{\mu\nu}, \quad (B1)
\]

where \( T(E)_{\mu\nu} \) and \( T(P)_{\mu\nu} \) are the energy-stress tensors for the electromagnetic wave and the related photons. Thus, Einstein [38] was wrong. In other words, Einstein's understanding on general relativity needs to be improved. Note that the energy-stress tensor of photons has an anti-gravity coupling. The presence of the photonic energy-stress is necessary; otherwise there is no bounded gravitational wave solution for equation (B1) [28, 30] and thus general relativity would be invalid.

In Einstein's initial assumption, the photons consist of only electromagnetic energy because general relativity has not been conceived. Since all the charged particles are massive, it is natural that the gravitational wave should be included. If the photons consist of only electromagnetic energy, there is a conflict since the photonic energy could be equivalent to mass and the electromagnetic energy-stress tensor is traceless. Now, this conflict is resolved since the photonic energy is the sum of electro-magnetic energy and gravitational energy.

Both quantum theory and relativity are required. This explains also that the absence of an anti-gravity coupling is the physical reason that the photons have no effect on the charged objects. Note that eq. (B2) was proposed by Lorentz [39] in 1916 and later by Levi-Civita [40] in the following form,

\[
G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -K [T(m)_{\mu\nu} - t(g)_{\mu\nu}], \quad (B2)
\]

where \( t(g)_{\mu\nu} \) is the energy-stress tensors for gravity. From (B2), the equation in vacuum is,

\[
G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = K t(g)_{\mu\nu}. \quad (B3')
\]

Note that \( t(g)_{\mu\nu} \) is equivalent to \( G^{(2)\mu\nu} \) in terms of Einstein's radiation formula.

When gravitational wave is present, the gravitational energy-stress tensor \( t(g)_{\mu\nu} \) is non-zero. Thus, a radiation of gravitational does carry energy-momentum as physics requires. This explains also that the absence of an anti-gravity coupling is the physical reason that the 1915 Einstein equation is incompatible with radiation [10-12].

Note that eq. (B2) was proposed by Lorentz [39] in 1916 and later by Levi-Civita [40] in the following form,

\[
\kappa t(g)_{\mu\nu} = G_{\mu\nu} + \kappa T_{\mu\nu} \quad (B4)
\]

where \( T_{\mu\nu} \) is the sum of other massive energy-stress tensors. Thus, it should be called the Lorentz-Lewin-Einstein equation although they have not proved its necessity. However, Einstein [41] rejected their proposal because he did not realize that, for his equation does not have any bounded dynamic solution [10-12].

Nevertheless, Princeton professors, Christodoulou and Klanerman have written a book [42] and claimed that they have constructed bounded dynamic solutions for the Einstein equation. Upon close examination, it is found [43] that they actually have not completed the proof for the existence of bounded dynamic solutions, in addition to other errors in mathematics [44]. If one wonders how such an error have happened, one must know that the thesis advisor of Christodoulou is J. A. Wheeler, whose mathematics is so inadequate that he made errors in their book [7] even at the undergraduate level [34].

**End Notes:**

1) For a thorough discussion on the relation between the mass and the total energy of a particle, one can read the 1989 paper of L. B. Okun [45]. However, Okun did not understand that the electromagnetic energy is not equivalent to mass [8] as shown in his 2008 paper [46].
2) Currently a major problem in general relativity is not only that Einstein’s errors are over-looked, but also that some theorists claimed as “experts” [7, 42, 47-50] who additionally make their own errors and ignore experimental facts because they cannot explain them.

3) Due to the popular opinion that gravity is attractive, Herrera, Santos, & Skea [51] argued that M in (2) involves the electric energy. They follow the error of Whittaker [52] and Tollman [53], and interpreted that M in the Reissner-Nordstrom metric includes the electromagnetic energy. Thus, according to their interpretation, an increase of the charge would lead to the increment of weight. Thus, the charge-mass interaction was over-looked. However, their interpretation is rejected by that a charged ball does reduce it weight [13]. Nevertheless, ‘t Hooft claimed in his Nobel Lecture [36] that the electric energy is part of the physical mass $m_{\text{phys}}$ of an electron. Moreover, he claimed this "physical mass" obeys Newton’s second law $F = m_{\text{phys}} a$. Note that such a claim violates special relativity because part of the electric energy is far from the electron and thus cannot react immediately as an inertial mass does. Another Nobel Laureate Wilczek [37] also incorrectly applied $m = E/c^2$ without providing a justification. His problem is that he cannot be distinct about the issue that a type of energy is equivalent to mass is different from that the mass is equivalent to energy.

4) In Newtonian theory, gravitation is always attractive to mass. However, in general relativity, the situation is different.

5) Theorists such a Misner et al. [7], Wald [47], Christodoulou & Klainerman [42], ‘t Hooft [54], and etc. claimed to have explicit dynamic solutions. It turns out that all are due to various errors in mathematics [26, 34, 55]. Note also that journals such a the Physical Review, Proceeding of the Royal Society, Classical and Quantum Gravity, Gravitation and General Relativity, etc. all are against Gullstrand [27] and believed incorrectly that there are bounded dynamic solutions for the Einstein equation [4, 10-12]. Another problem is that the editors would not read papers that disagree with them.

6) A well-known misleading result is the positive mass theorem of Schoen & Yau [49] (and the positive energy theorem of Witten [50]). Their failure in understanding general relativity properly could be a reason that the lack of progress of the string theory in physics. They were awarded Fields Medals in 1982 and in 1990 because leading mathematicians such M. F. Atiyah do not understand the related physics [29].

7) I have reported these problems in $E = mc^2$ to MIT President Hockfield and the subsequent President Reif. They have promised to upgrade the related education in gravitation.

8) Theorists such as Hod [56] invalidly claimed to have “A simplified two-body problem in general relativity” because they do not understand non-linear mathematics. They did not know that the linearized equation is actually a linearization of the 1995 Lorentz-Levy-Einstein equation up-dated [10], but is not valid of the Einstein equation.

9) The 2011 Shaw Prize also made a mistake by awarding a half prize to Christodoulou for his work, based on obscure errors, against the honorable Gullstrand [27] Chairman (1922-1929) of the Nobel Committee for Physics. Prof. Christodoulou has misled many including the 1993 Nobel Committee [35]. However, his errors are now well-established since they have been illustrated with mathematics at the undergraduate level [31]. Christodoulou claimed in his autobiography that his work is based on two sources: 1) The claims of Christodoulou and Klainerman on general relativity as shown in their book The Global Nonlinear Stability of the Minkowski Space [42]; 2) Roger Penrose had introduced, in 1965, the concept of a trapped surface and had proved that a space-time containing such a surface cannot be complete [31]. However, this work of Penrose, which uses an implicit assumption of unique sign for all coupling constants, actually depends on the errors of Christodoulou and Klainerman [10, 43].

References:

6. Takaaki Musha, “Theoretical explanation of the Biefeld-Brown Effect”, 3-11-7-601 Namiki, Kanazawa-ku, Yokohama 236-0005 Japan, E-mail: musha@jda-trdi.go.jp.
13. D. Yu. Tsipenyuk, V. A. Andreev, Physical Interpretations of the Theory of Relativity Conference, Bauman Moscow State Technical University, 4 – 7 July 2005. e-mail: andrvlad@yandex.ru.
34. C. Y. Lo, Errors of the Wheeler School, the Distortions to General Relativity and the Damage to Education in MIT Open Courses in Physics, GJSFR Vol. 13 Issue 7 Version 1.0 (2013).