ON ALMOST $C(\alpha)$-MANIFOLD SATISFYING SOME CONDITIONS ON THE WEYL PROJECTIVE CURVATURE TENSOR

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ABSTRACT. In the present paper, we have studied the curvature tensors of almost $C(\alpha)$-manifolds satisfying the conditions $P(\xi, X)R = 0$, $P(\xi, X)\widetilde{Z} = 0$, $P(\xi, X)P = 0$, $P(\xi, X)S = 0$ and $P(\xi, X)\widetilde{C} = 0$. According these cases, we classified almost $C(\alpha)$-manifolds.

1. INTRODUCTION

In [10], authors studied the Weyl projective curvature tensor in an $N(k)$-contact metric manifold and classified $N(k)$-contact metric manifolds.

In [3] and [9], we searched the properties of curvature tensors of an almost $C(\alpha)$-manifold satisfying $\widetilde{Z}(\xi, X)R = \widetilde{Z}(\xi, X)\widetilde{Z} = \widetilde{Z}(\xi, X)S = \widetilde{Z}(\xi, X)P = 0$ and Ricci semi-symmetric, projective semi-symmetric, quasi-conformal semi-symmetric.

De U. C. and Sarkar A. [4] studied properties of projective curvature tensor to generalized Sasakian space form. Atçeken M. [2] studied generalized Sasakian space form satisfying certain conditions on the concircular curvature tensor. Özgür M. and De U. C. [6] researched some certain curvature conditions satisfied by quasi-conformal curvature tensor in Kenmotsu manifolds. Arslan K., Murathan C. and Özgür C. produced the works on contact manifold curvature tensor[1].

Motivated by the studies of the above authors, in this paper we classify almost $C(\alpha)$-manifolds, which satisfy the curvature conditions $P(\xi, X)R = 0$, $P(\xi, X)\widetilde{Z} = 0$, $P(\xi, X)P = 0$, $P(\xi, X)S = 0$ and $P(\xi, X)\widetilde{C} = 0$, where P is the Weyl projective curvature tensor, $\widetilde{Z}$ is the concircular curvature tensor, S is the Ricci tensor and $\widetilde{C}$ is quasi-conformal curvature tensor.

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2. Preliminaries

An odd-dimensional Riemannian manifold (M, g) is said to be an almost co-Hermitian or almost contact metric manifold if there exist on M a $(1,1)$-tensor field ϕ, a vector field ξ (called the structure vector field) and a 1-form η such that

$$\eta(\xi) = 1, \qquad \phi^2 X = -X + \eta(X)\xi, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.2}$$

$$\phi\xi = 0, \qquad \eta o \phi = 0, \tag{2.3}$$

for any vector field X, Y on M.

The Sasaki form (or fundamental 2-form) Φ of an almost co-Hermitian manifold (M, g, ϕ, ξ, η) is defined by

$$\Phi(X, Y) = g(X, \phi Y)$$

for all X, Y on $\in \chi(M)$ and this form satisfies $\eta \wedge \Phi^n \neq 0$. This means that every almost co-Hermitian manifold is orientable and (η, Φ) defines an almost cosymplectic structure on M. If this associated structure is cosymplectic ($d\Phi = d\eta = 0$), M is called an almost co-Kähler manifold. The associated almost cosymplectic structure is a contact structure and is an almost Sasakian manifold when $\Phi = d\eta$. It is well known that every contact manifold has an almost Sasakian structure.

The Nijenhuis tensor of the (1,1)-tensor field ϕ is the (1,2)-tensor field $[\phi, \phi]$ defined by

$$[\phi, \phi](X, Y) = \phi^2[X, Y] + [\phi X, \phi Y] - \phi[\phi X, Y] - \phi[X, \phi Y], \tag{2.4}$$

where $[X, Y]$ is the Lie bracket of $X, Y \in \chi(M)$.

On the other hand, an almost co-complex structure is called integrable if $[\phi, \phi] = 0$ and normal if $[\phi, \phi] + 2d\eta \otimes \xi = 0$. A co-Kähler manifold (or normal cosymplectic manifold) is an integrable (or equivalently, a normal) almost contact Kähler manifold, while a Sasakian manifold is a normal almost Sasakian manifold[5].

The Riemannian connections ∇ of Sasakian, co-Kähler and Kenmotsu manifolds have some well known properties which allow us to characterize these manifolds.

Theorem 2.1. *Let* (M, g, ϕ, ξ, η) *be an almost co-Hermitian manifold with Riemannian connection* ∇*. Then*

(i) M *is co-Kählerian if and only if* $\nabla\phi = 0$,

(ii) M *is Sasakian if and only if*

$$(\nabla_X \phi)Y = g(X, Y)\xi - \eta(Y)X,$$

(iii) M *is Kenmotsu manifold if and only if*

$$(\nabla_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X.$$

for all $X, Y \in \chi(M)$[5].

Theorem 2.2. ξ *is Killing vector field for co-Kähler and Sasaki manifolds, i.e.*

$$g(\nabla_X \xi, Y) + g(X, \nabla_Y \xi) = 0,$$

while for Kenmotsu manifolds we have

$$g(\nabla_X \xi, Y) - g(X, \nabla_Y \xi) = 0.$$

for all $X, Y \in \chi(M)$[5].

Theorem 2.3. *Let* R *be the Riemann curvature tensor on* M*. For all* $X, Y, Z, W \in \chi(M)$*, we have*

(i) for M *co-Kählerian:*

$$R(X, Y, Z, W) = R(X, Y, \phi Z, \phi W);$$

(ii) for M *Sasakian:*

$$\begin{aligned} R(X, Y, Z, W) &= R(X, Y, \phi Z, \phi W) - g(X, Z)g(Y, W) + g(X, W)g(Y, Z) \\ &+ g(X, \phi Z)g(Y, \phi W) - g(X, \phi W)g(Y, \phi Z); \end{aligned}$$

(iii) for a Kenmotsu manifold M*:*

$$\begin{aligned} R(X, Y, Z, W) &= R(X, Y, \phi Z, \phi W) + g(X, Z)g(Y, W) - g(X, W)g(Y, Z) \\ &- g(X, \phi Z)g(Y, \phi W) + g(X, \phi W)g(Y, \phi Z), \end{aligned}$$

Definition 2.4. An almost $C(\alpha)$-manifold M is an almost co-Hermitian manifold such that the Riemann curvature tensor satisfies the following property: $\exists \alpha \in R$ such that

$$\begin{aligned} R(X, Y, Z, W) &= R(X, Y, \phi Z, \phi W) + \alpha\{-g(X, Z)g(Y, W) + g(X, W)g(Y, Z) \\ &+ g(X, \phi Z)g(Y, \phi W) - g(X, \phi W)g(Y, \phi Z)\}. \end{aligned} \tag{2.5}$$

for all $X, Y, Z, W \in \chi(M)$.

Moreover, if such a manifold has constant ϕ-sectional curvature equal to c, then its curvature tensor is given by

$$\begin{aligned} R(X,Y)Z &= (\frac{c+3\alpha}{4})\{g(Y,Z)X - g(X,Z)Y\} \\ &+ (\frac{c-\alpha}{4})\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\} \\ &+ (\frac{c-\alpha}{4})\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi \\ &- g(Y,Z)\eta(X)\xi\}. \end{aligned} \tag{2.6}$$

A normal almost $C(\alpha)$-manifold is called $C(\alpha)$-manifold[5].

Co-Kählerian, Sasakian and Kenmotsu manifolds are, respectively, $C(0)$, $C(1)$ and $C(-1)$-manifolds.

Theorem 2.5. *An almost co-Hermitian manifold M is α-Sasakian if and only if for all $X, Y \in \chi(M)$*

$$(\nabla_X \phi)Y = \alpha\{g(X,Y)\xi - \eta(X)Y\}. \tag{2.7}$$

(ii) If M is α-Sasakian, then ξ is a Killing vector field and

$$\nabla_X \xi = -\alpha\phi X \tag{2.8}$$

for all $X \in \chi(M)$.
(iii) An α-Sasakian manifold is a $C(\alpha^2)$-manifold[5].

Theorem 2.6. *An almost co-Hermitian manifold is an α-Kenmotsu manifold if and only if*

$$(\nabla_X \phi)Y = \alpha\{g(\phi X,Y)\xi - \eta(Y)\phi X\}, \tag{2.9}$$

$$\nabla_X \xi = \alpha\{-X + \eta(X)\xi\}, \tag{2.10}$$

for all $X, Y \in \chi(M)$.

(ii) An α-Kenmotsu manifold is a $C(-\alpha^2)$-manifold[5].

The concept of quasi-conformal curvature tensor was defined by K. Yano and S. Sawaki [8]. Quasi-conformal curvature tensor of a $(2n+1)-$dimensional Riemannian manifold is defined as

$$\begin{aligned} \widetilde{C}(X,Y)Z &= aR(X,Y)Z + b[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX \\ &- g(X,Z)QY] - \frac{r}{2n+1}[\frac{a}{2n} + 2b][g(Y,Z)X - g(X,Z)Y], \end{aligned} \tag{2.11}$$

where, a and b are arbitrary constants, Q, S and r denote the Ricci operator, Ricci tensor and scalar curvature of manifold, respectively. If $\widetilde{C} = 0$, then manifold is said to be quasi-conformal flat.

Let M be $(2n+1)-$dimensional Riemannian manifold. The Weyl projective curvature tensor field is defined by [7]

$$P(X,Y)Z = R(X,Y)Z - \frac{1}{2n}[S(Y,Z)X - S(X,Z)Y], \tag{2.12}$$

for any $X, Y, Z \in \chi(M)$.

Let (M, g) be an $(2n + 1)$-dimensional Riemannian manifold. Then the concircular curvature tensor $\widetilde{Z}$ is defined by

$$\widetilde{Z}(X,Y)Z = R(X,Y)Z - \frac{r}{2n(2n+1)}(g(Y,Z)X - g(X,Z)Y), \tag{2.13}$$

for all $X, Y, Z \in \chi(M)$, where r is the scalar curvature of M[7].

3. An almost $C(\alpha)$-Manifold Satisfying Certain Conditions on the Weyl Projective Curvature Tensor

In this section, we will give the main results for this paper.

Let M be $(2n + 1)$-dimensional almost $C(\alpha)$-manifold and we denote the Riemannian curvature tensor of R, then we have from (2.6), for $X = \xi$,

$$R(\xi,Y)Z = \alpha\{g(Y,Z)\xi - \eta(Z)Y\}. \tag{3.1}$$

In the same way, choosing $Z = \xi$ in (2.6), we have

$$R(X,Y)\xi = \alpha\{\eta(Y)X - \eta(X)Y\}. \tag{3.2}$$

In (3.2), choosing $Y = \xi$, we obtain

$$R(X,\xi)\xi = \alpha\{X - \eta(X)\xi\}. \tag{3.3}$$

Also, from (2.6), we obtain

$$\eta(R(X,Y)Z) = \alpha\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\}. \tag{3.4}$$

In the same way choosing $X = \xi$ in (2.11), we have

$$\begin{aligned}\widetilde{C}(\xi,Y)Z &= \{a\alpha + 2n\alpha b - \frac{r}{2n+1}[\frac{a}{2n} + 2b]\}\{g(Y,Z)\xi - \eta(Z)Y\} \\ &+ b\{S(Y,Z)\xi - \eta(Y)QY\}.\end{aligned} \tag{3.5}$$

In (3.5), choosing $Z=\xi$, we obtain

$$\begin{aligned}\widetilde{C}(\xi,Y)\xi &= \{a\alpha+2n\alpha b-\frac{r}{2n+1}[\frac{a}{2n}+2b]\}\{\eta(Y)\xi-Y\}\\ &+ b\{2n\alpha\eta(Y)\xi-QY\}.\end{aligned}\tag{3.6}$$

Also, from (2.13) we have

$$\widetilde{Z}(\xi,X)Y=\{\alpha-\frac{r}{2n(2n+1)}\}\{g(X,Y)\xi-\eta(Y)X\}\tag{3.7}$$

and

$$\widetilde{Z}(\xi,X)\xi=\{\alpha-\frac{r}{2n(2n+1)}\}\{\eta(X)\xi-X\}.\tag{3.8}$$

Also, from (2.12), we have

$$P(\xi,Y)Z=\alpha g(Y,Z)\xi-\frac{1}{2n}S(Y,Z)\xi.\tag{3.9}$$

From (2.6), we can state

$$\begin{aligned}R(X,e_i)e_i+R(X,\phi e_i)\phi e_i+R(X,\xi)\xi &= \sum_{i=1}^{n}\{(\frac{3\alpha+c}{4})\{nX-g(X,e_i)e_i+nX\\ &- g(X,\phi e_i)\phi e_i+X-g(X,\xi)\xi\}\\ &+ (\frac{c-\alpha}{4})\{3g(X,\phi e_i)\phi e_i-2n\eta(X)\xi\\ &+ 3g(X,\phi^2 e_i)\phi^2 e_i\eta(X)\xi-X\}\},\end{aligned}\tag{3.10}$$

for $\{e_1,e_2,...,e_n,\phi e_1,...,\phi e_n,\xi\}$ orthonormal basis of M. From (3.10), for $Y\in\chi(M)$, we obtain

$$\begin{aligned}S(X,Y) &= \left(\frac{\alpha(3n-1)+c(n+1)}{2}\right)g(X,Y)\\ &+ \left(\frac{(\alpha-c)(n+1)}{2}\right)\eta(X)\eta(Y),\end{aligned}\tag{3.11}$$

which is equivalent to

$$QX=\left(\frac{\alpha(3n-1)+c(n+1)}{2}\right)X+\left(\frac{(\alpha-c)(n+1)}{2}\right)\eta(X)\xi.\tag{3.12}$$

From (3.11), we can give the following corollary.

Also, from (3.11), we can easily see

$$r=n[\alpha(3n+1)+c(n+1)],\tag{3.13}$$

$$S(X,\xi)=2n\alpha\eta(X),\tag{3.14}$$

and

$$Q\xi = 2n\alpha\xi. \tag{3.15}$$

Theorem 3.1. *Let M be (2n+1)-dimensional an almost $C(\alpha)$-manifold. Then, $P(\xi, X)R = 0$ if and only if M reduce real space form with constant sectional curvature c.*

Proof. Suppose that $P(\xi, X)R = 0$. Then, we have

$$\begin{aligned}(P(\xi,X)R)(U,W)Z &= P(\xi,X)R(U,W)Z - R(P(\xi,X)U,W)Z \\ &- R(U,P(\xi,X)W)Z - R(U,W)P(\xi,X)Z \\ &= 0. \end{aligned} \tag{3.16}$$

Using (3.9) in (3.16), we obtain

$$\begin{aligned} &= \alpha\{g(X,R(U,W)Z)\xi - g(X,U)R(\xi,W)Z \\ &- g(X,W)R(U,\xi)Z - g(X,Z)R(U,W)\xi\} \\ &- \frac{1}{2n}\{S(X,R(U,W)Z)\xi - S(X,U)R(\xi,W)Z \\ &- S(X,W)R(U,\xi)Z - S(X,Z)R(U,W)\xi\} \\ &= 0. \end{aligned} \tag{3.17}$$

Putting $U = \xi$ in (3.17) and using the equations (3.1) and (3.2), we have

$$\begin{aligned}\frac{1}{2n}S(X,W)\eta(Z) &= \alpha\{g(X,W)\eta(Z) + \eta(Z)\eta(X)\eta(W) \\ &- g(W,Z)\eta(X)\}, \end{aligned} \tag{3.18}$$

which implies that

$$S(X,W) = 2n\alpha g(X,W).$$

So, the almost $C(\alpha)$-manifold is an Einstein manifold. In this case $r = 2n\alpha(2n+1)$. Taking into account of (3.13), we obtain $\alpha = c$, which implies that

$$R(X,Y)Z = c\{g(Y,Z)X - g(X,Z)Y\}.$$

The converse is obvious. □

Theorem 3.2. *Let M be (2n+1)-dimensional an almost $C(\alpha)$-manifold. Then, $P(\xi, X)\widetilde{Z} = 0$ if and only if M is a real space form with sectional curvature c.*

Proof. Suppose that $P(\xi, X)\widetilde{Z} = 0$, we have

$$\begin{aligned}(P(\xi,X)\widetilde{Z})(U,W)Z &= P(\xi,X)\widetilde{Z}(U,W)Z - \widetilde{Z}(P(\xi,X)U,W)Z \\ &- \widetilde{Z}(U,P(\xi,X)W)Z - \widetilde{Z}(U,W)P(\xi,X)Z \\ &= 0. \end{aligned} \tag{3.19}$$

Using (2.13) and (3.9) in (3.19), we obtain

$$\begin{aligned} 0 &= \alpha\{g(X,\widetilde{Z}(U,W)Z)\xi - g(X,U)\widetilde{Z}(\xi,W)Z - g(X,W)\widetilde{Z}(U,\xi)Z \\ &- g(X,Z)\widetilde{Z}(U,W)\xi\} - \frac{1}{2n}\{S(X,\widetilde{Z}(U,W)Z)\xi - S(X,U)\widetilde{Z}(\xi,W)Z \\ &- S(X,W)\widetilde{Z}(U,\xi)Z - S(X,Z)\widetilde{Z}(U,W)\xi\}. \end{aligned} \tag{3.20}$$

In (3.20), choosing $U=\xi$ and using (2.13), (3.7), (3.8) and (3.14), we have

$$\begin{aligned} 0 &= [\alpha - \frac{r}{2n(2n+1)}]\{\alpha g(X,Z)W - \alpha g(X,Z)\eta(W)\xi \\ &- \alpha g(X,W)\eta(Z)\xi + \frac{1}{2n}S(X,W)\eta(Z)\xi + \frac{1}{2n}S(X,Z)\eta(W)\xi \\ &- \frac{1}{2n}S(X,Z)W\}. \end{aligned} \tag{3.21}$$

Inner product both sides of the equation by ξ, we have □

$$[\alpha - \frac{r}{2n(2n+1)}]\{\frac{1}{2n}S(X,W) - \alpha g(X,W)\} = 0$$

If $r = 2n\alpha(2n+1)$, from (3.13), we obtain $\alpha = c$. This implies that M is a real space form. Otherwise $S(X,Y) = 2n\alpha g(X,Y)$. This tells us $r = 2n\alpha(2n+1)$.

Theorem 3.3. *Let M be (2n+1)-dimensional an almost $C(\alpha)$-manifold.Then, $P(\xi,Y)P = 0$ if and only if M reduce real space form with constant sectional curvature $c=\alpha$.*

Proof. Suppose that $P(\xi,Y)P$=0, we have

$$\begin{aligned} (P(\xi,Y)P)(Z,U)W &= P(\xi,Y)P(Z,U)W - P(P(\xi,Y),U)W \\ &- P(Z,P(\xi,Y)U)W - P(Z,U)P(\xi,Y)W \\ &= 0. \end{aligned} \tag{3.22}$$

Using (3.9) in (3.22), we have

$$\begin{aligned} 0 &= \alpha\{g(Y,P(Z,U)W)\xi - \alpha g(Y,Z)g(U,W)\xi + \frac{1}{2n}g(Y,Z)S(U,W)\xi \\ &- \frac{1}{2n}g(Y,U)S(Z,W)\xi + \alpha g(Y,U)g(W,Z)\xi\} \\ &+ \frac{1}{2n}\{-S(Y,P(Z,U)W)\xi + \alpha g(U,W)S(Y,Z)\xi - \frac{1}{2n}S(Y,Z)S(U,W)\xi \\ &+ \frac{1}{2n}S(Y,U)S(Z,W)\xi - \alpha S(Y,U)g(W,Z)\xi\}. \end{aligned} \tag{3.23}$$

Using the equations (2.12) and (3.11) in (3.23), we obtain

$$[\frac{(\alpha-c)(n+1)}{4n}]\big[R(Z,U)W - \alpha\{g(U,W)Z - g(W,Z)U\}\big] = 0,$$

which proves our assertion. □

Theorem 3.4. *Let M be (2n+1)-dimensional an almost $C(\alpha)$-manifold. Then, $P(\xi, Y)\widetilde{C} = 0$ if and only if M has either α-sectional curvature or it is an Einstein manifold.*

Proof. Suppose that $P(\xi, Y)\widetilde{C} = 0$, we have

$$\begin{aligned}(P(\xi, Y)\widetilde{C})(Z, U)W &= P(\xi, Y)\widetilde{C}(Z, U)W - \widetilde{C}(P(\xi, Y)Z, U)W \\ &- \widetilde{C}(Z, P(\xi, Y)U)W - \widetilde{C}(Z, U)P(\xi, Y)W \\ &= 0. \end{aligned} \tag{3.24}$$

Using (3.9) in (3.24), we obtain

$$\begin{aligned} 0 &= \alpha\{g(Y, \widetilde{C}(Z, U)W)\xi - g(Y, Z)\widetilde{C}(\xi, U)W \\ &- g(Y, U)\widetilde{C}(Z, \xi)W - g(Y, W)\widetilde{C}(Z, U)\xi\} \\ &- \frac{1}{2n}\{S(Y, \widetilde{C}(Z, U)W)\xi - S(Y, Z)\widetilde{C}(\xi, U)W \\ &- S(Y, U)\widetilde{C}(Z, \xi)W - S(Y, W)\widetilde{C}(Z, U)\xi\} \\ &= 0. \end{aligned} \tag{3.25}$$

In (3.25), choosing $Z = \xi$ and using (3.5) and (3.6), we obtain

$$\begin{aligned} 0 &= \alpha\{a\alpha + 2n\alpha b - \frac{r}{2n+1}[\frac{a}{2n} + 2b]\}\{g(Y, QU) - 2n\alpha g(Y, U)\} \\ &+ b\{S(Y, QU) - S(U, Y)\} \end{aligned} \tag{3.26}$$

Using (3.12) in (3.26) and choosing $U = \phi U$, we have

$$[\frac{(n+1)(c-\alpha)}{2}]\{bS(\phi U, Y) + [a\alpha + 2n\alpha b - \frac{r}{2n+1}[\frac{a}{2n} + 2b]]g(\phi U, Y)\} = 0.$$

The proof is completed. □

Theorem 3.5. *Let M be (2n+1)-dimensional an almost $C(\alpha)$-manifold. Then, $P(\xi, X)S = 0$ if and only if M is an Einstein manifold.*

Proof. Suppose that $P(\xi, X)S = 0$, we have

$$S(P(\xi, X)U, W) + S(U, P(\xi, X), W) = 0. \tag{3.27}$$

In (3.27), using (3.9), we have

$$\alpha\{g(X, W)\xi + g(X, U)\xi\} - \frac{1}{2n}\{S(X, W)\xi + S(X, U)\xi\} = 0 \tag{3.28}$$

Inner product both sides of (3.28) by $\xi \in \chi(M)$, and choosing $U = \xi$, we have

$$S(X, W) = 2n\alpha g(X, W).$$

So, M is an Einstein manifold. □

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