ARE THETA FUNCTIONS THE FOUNDATIONS OF PHYSICS?

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ABSTRACT

The Jacobi theta functions are essentially rotations in a complex space and as such provide a basis for the lattices of the exceptional Lie algebras $E_6, E_8$ in complex 3-space and complex 4-space. In this note we will show that a choice of the nome $q$ of the theta functions $\theta_{E_6}, \theta_{E_8}$ leads to the equilateral tritangents of these lattices. Specifically we will find quarter period ratios of the real and complex axes.

In particular $E_8$ is isomorphic to the binary icosahedral group shown recently to describe Elementary Particle theory and Quantum Gravity so $q$ is fundamental to the structure of space itself.

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1 INTRODUCTION

This note should be read as a supplement to a recent publication[3] where it was shown how quantum gravity and icosahedral symmetry both employ the same Jacobi Theta function

\[ \theta_{E_8} = 1 + 240q^2 + 2160q^4 + 6720q^6 + \ldots \]  

(1)

where \( N_m = 240 \) is the number of vertices in the 8 successive shells of the toric lattice shown in Fig.2 and \( q = \exp(i\pi t) \) is the elliptic nome \( \exp(-\pi iK/K) \) with a maximum value of 0.06586 [5], where K and iK = K' are quarter periods. \( N_m = 240 \) is also the kissing number for a sphere packing of the \( E_8 \) lattice in Fig.2, where the binary icosahedral group is isomorphic to the exceptional Lie algebra \( E_8 \) by the MacKay correspondence[7]. Here there are 8 sets of 30 vertices on 4 dual Riemann surfaces but the third coefficient \( N_m = 2160 \) is no longer a kissing number between spheres because it belongs to another set of vectors given in ([1], Table 4.10). But \( N_m \) remains the number of vectors in successive shells of the toric lattice.

In this contribution we will find that the ratio iK/K in the elliptic nome is \( \sqrt{3}/2 = \sin 120^\circ \) and is the same for the theta functions of the exceptional Lie algebras \( E_6 \subset E_8 \) and therefore determines the geometry of the equilateral tritangents appearing in Figs.1, 2. In this way theta functions determine icosahedral symmetry and the structure of space underlining particle physics and quantum gravity [3].

2 Jacobi Theta Functions

We will employ the Jacobi theta function

\[ \theta = \sum_{m=-\infty}^{\infty} \exp(i\pi m^2 + 2i\pi z) \]  

(2)

for the \( \theta_{E_8}, \theta_{E_6}, \theta_{E_6} \) lattices([1],[4]) where \( \exp(i\pi \tau) = q \) and the dependence on \( z \) is carried by the lattice \( \Gamma \) so (2) reduces to

\[ \theta = \sum_{m=1}^{\infty} r_\gamma(2m)q^{2m} \]  

(3)

where \( r_\gamma \) is the kissing number \( N_m = 240 \) in equation (1) according to Lewis Lewark[6]. Then the theta series for \( E_6 \) is[1] Ch.4

\[ \theta_{E_6} = 1 + 72q^2 + 270q^4 + 720q^6 + \ldots \]  

(4)

Fig.1 shows 12 vertices on 2 Riemann surfaces plus 3 at the origin thus accounting for 27 elementary particles of the Standard Model. Only 27 of 72 vertices rotate into themselves by \( \omega = 120^\circ \) degrees and Coxeter([2] p.119) labels these by \((0, \omega^3, -\omega), (-\omega, 0, 1)\) in the equilateral tritangents Fig.12.3A which should be replaced by the torus of Fig.1 according to [3].

However if \( q = \exp(i\pi K/K) = 0.0658 \) then it follows that iK/K = \( \sqrt{3}/2 = \sin \omega \) and it may easily be shown that this value of \( q \) remains the same for the powers in equations (1),(4). The remaining terms of the tori(1),(4) are multiples of the kissing number possibly implying quantum entanglement.

3 Conclusion

We have seen how \( \sin \omega = \sin 120^\circ \) specifies the tritangents in the lattices of \( E_6, E_8 \). Specifically in Section 12.5 of[2] Coxeter associates \( E_8 \) with the Witting Polytrope shown in the frontispiece and Fig.2. Here there are 27 edges, labeled by \( \omega \), at each vertex which implies quantum entanglement and illustrates how theta functions underlie the elementary particles and nucleons.

There are only 3 complex axes \( i, j, k \) which are a basis for the quaternion algebra and \( E_8 \) is isomorphic to the binary octahedral group by the MacKay correspondence [7] and is the largest of the Exceptional Lie algebras. There are no more.
Fig.1 Graph of $E_6$

Fig.2 Graph of $E_8$

References